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ENTROPY LAW, MAXWELL'S DEMON AND QUASI-TWO-DIENSIONAL TERBULENCE

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Abstract: Two or Quasi-two dimensional turbulence has two conserved quantities, the enstrophy (squared vorticity) in addition to the energy. As a result, it has a unique property in which the energy spectrum cascades to small wave numbers (often called inverse cascade), while the enstrophy spectrum to large wave numbers. Owing to the inverse cascade, simulation of turbulence of two-dimensional fluids demonstrates formation of larger structures in energy contours clearly forming more ordered (or self organized) structure. Question then arises as to what happens to the system entropy. Does the entropy decrease in such a system in contradiction to the second law of thermodynamics? In this paper, based on a simulation result of quasi-two-dimensional plasma turbulence I propose a process in which this dilemma is resolved. Here, the entropy-decreasing region is geographically divided across a demarcation layer beyond which the entropy continues to increase. The overall entropy may still increase in accordance with the second law of thermodynamics but entropy may decrease locally in a region of geographically divided region. Such local or regional decrease of entropy is in fact possible without violating the second law, provided that overall entropy of the system increases. Furthermore it seems as though the Maxwell demon is operating at the demarcation layer that differentiates low and high entropy regions. The inverse cascade and regional decrease of plasma entropy presents a new criterion for ignition of controlled fusion. Instead of energy confinement time, confinement of negentropy state should replace the Lawson criterion.

TWO-DIMENSIONAL FLUID TURBULENCE

Turbulence excited in two-dimensional fluids is known to have a unique property in which the energy spectrum cascades to small wavenumbers, contrary to three-dimension turbulence. This is a consequence of existence of two conserved quantities, the energy and a quantity called the enstrophy, which is the squared vorticity. As a result Kraichnan (1967) showed that there are two inertial range in turbulence spectrum, one for energy v_k^2 and the other for enstrophy, $(\mathbf{k}\times\mathbf{v_k})^2$, where, $\mathbf{v_k}$ is the Fourier

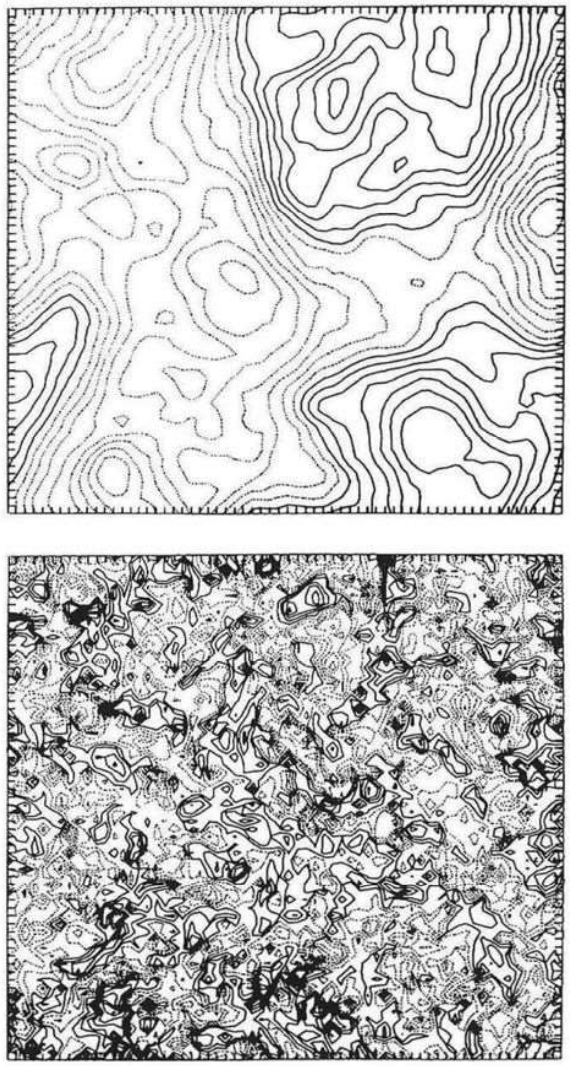

Fig.1 Simulaiton results of two-dimensionalhydrodynamic turbulence obtainedby Lilly (1969).

amplitude of the velocity field. In the inertial range of energy spectrum, the energy cascades to small wave numbers (and enstrophy spectrum has no cascade), while in that of enstrophy, the enstrophy spectrum cascadesto large wavenumbers. Because of this nature, such cascade process is called the dual cascade of turbulence spectra. Simulation result shown below of two-dimensional hydrodynamics turbulence obtained by Lilly (1969) clearly demonstrates the nature of the dual cascade; formation of large structures in stream function, i.e., energy contour, while the vorticity forms turbulent small-scale structures. The process clearly demonstrates decreasing entropy as defined by log of probability distribution of wavenumber spectrum of the energy, while entropy defined by probability distribution of enstrophy increases.

The simulation result is interesting, but since the two dimensional fluids does not exists in reality the self-organization and resulting entropy decrease seems somewhat academic.

QUISI-TWO-DIMENSIONAL PLASMA AND ATMOSPHERIC TURBULENCE

Electrostatic turbulence in magnetized plasma as described by Hasegawa-Mima equation (Hasegawa and Mima, 1978) exhibits a nature similar to that of two-dimensional hydrodynamics because of existence of two conserved quantities, energy and potential enstrophy (sum of squared vorticity and potential energy). The model equation allows three-dimensional motion of electrons and as a result it is more physical than the ideal two-dimensional hydrodynamics of Kraichnan and Lilly model.

Besides, because of three dimensional motion of electrons, the fluid motion becomes compressible, i. e, unlike the two dimensional fluids of Kraichnan, divergence of the velocity field is not zero that indicates the existence of sink or source. The Hasegawa-Mima equation has a structure quite analogous to that which describes hydrodynamic turbulence of atmospheric motion on rotating planets where the vertical motion of the air is assumed to have static balance with the gravity (Rossby 1939). The difference is that Hasegawa-Mima equation takes into account of plasma density gradient, while the Rossby equation the gradient of Coriolis parameter. But the important analogy is the existence of divergence of velocity field.

Recently the self-organized states of turbulence that result as a consequence of the dual cascade in TOKAMAK plasmas are believed to form zonal flow that inhibits the anomalous heat transport toward the edge and helps plasma confinements (i. e., Diamond et al 2011), and thus the concept of inverse cascade and subsequent self-organization is attracting interest in practical systems. As a matter of fact, Hasegawa and Wakatani (1987) first demonstrated the formation of zonal flow as a result of quasi-two dimensional turbulence. as shown in the figure below.

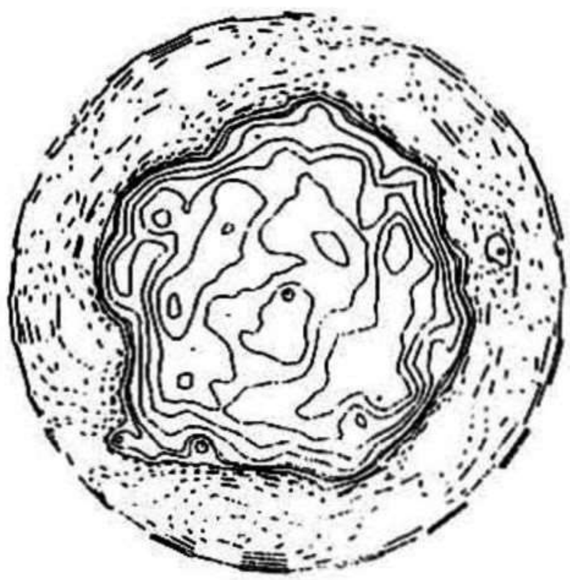

Fig.2 Simulation result of plasma turbulence obtained by Hasegawa and Wakatani (1987)

I have revisited the simulation result of Hasegawa and Wakatani (1987) and identified several interesting properties of the turbulence, not identified in this paper.

First, analogous to the atmospheric vortices, ion vortices having positively (negatively) charged cores tend to expand (contract) their size. In atmosphere, the high pressure eddies tend to expand forming extended good weather region, while low pressure eddies to shrink and become typhoon or hurricane. Therefore the dynamical properties of the negative and positive vortices are asymmetric, unlike those in two-dimensional fluids. This asymmetry seems to be the consequence of the centrifugal force of rotating ions being parallel (antiparallel) to the electrostatic force (or pressure gradient force in the atmosphere) of the positive (negative) core charge. Note, however, that in the ideal two-dimensional plasma, the circular motion is due to **EXB** drift, which is not compressible and mass independent thus there is no centrifugal force. However, in the quasi-two dimensional fluid described here, the fluid is compressible and the centrifugal force appears due to the polarization drift of ions. Combination of time varying sink or source and the polarization drift induces spiral motion of fluids resulting in asymmetric fluid dynamics.

Second, the positively charged vortices tend to gather in the central region of the cylindrical plasma, where the plasma density is maximum, while negatively charged vortices tend to gather near the edge regions. As a result the size of vortices in the central (edge) region tend to expand (shrink) their size As identified by Hasegawa and Wakatani, the results of the turbulence produces zero equipotential line that is closed near the plasma edge. The potential profile creates the zonal flow of the plasma in the azimuthal direction.

Third, there seems clear regional distinction in the way the entropy is produced. In the central region the entropy seems to decrease, while in the edge region it increases. As a result it seems that the zero potential line that differentiate the two regions is acting to differentiate the way the entropy is produced. More specifically, unlike the simulation result of Lilly for two-dimensional fluids where the global entropy defined by energy spectrum decreases, in the quasi-two-dimensional plasma turbulence there exists clear regional distinction in the entropy production. It seems as though the Maxwell demon is sitting at the regional boundary and acting to differentiate the entropy production in such a way the entropy in the central region tend to decrease while that of the edge increase.

Furthermore, the simulation result clearly illustrates the physical meaning of the Kraichnan's dual cascade, the inverse cascades results from the increase of the size of positively charged vortices but it takes place only regionally in the central region, not globally. There exists regionally normal cascading area near the plasma edge, whereby the global increase of entropy is taken care of; there exists no physical mystery.

IMPLICATION OF LOCAL REDUCTION OF ENTROPY

The fact the plasma entropy may decrease regionally near the core region has an interesting implication in plasma confinement. It is compatible with the increase of energy content per flux tube near the core, which is favorable for energy confinement. Heating plasma by neutral beam or RF near the core injects negentropy (energy source having lower entropy than that of the plasma as a whole owing to monochromatic energy spectrum of neutral beam or RF) that further acts to decrease the regional entropy that helps plasma profile confinement. Since the injected energy diffuses to the edge and lost, what is essential for fusion to be sustained is not the energy confinement as classically identified as Lawson criteria, but confinement of the plasma pressure profile, or negentropy state which is maintained by the regional decrease of entropy by the process shown here. I believe the line of thought presented here, turbulent decrease of entropy near the core, or creation of negentropy there and subsequent production of favorable pressure profile present revolutionary new perception of plasma confinement for the purpose of nuclear fusion. What is needed for an operation of fusion plasma is not to achieve sufficiently long energy confinement as classically believed, but to achieve sufficiently long small entropy (or negentropy) state. The inverse cascade nature of the plasma turbulence may allow achieving such a condition. While if the energy is not confined, but if the negentropy state is maintained by providing proper pressure gradient,

$\partial p / \partial \psi < 0$,

the fusion reactor works as energy amplifier, which is acceptable.

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（通信　昭和32年卒　34年修士）